A Parametric Study of the SASI Comparing General Relativistic and Non-Relativistic Treatments

Samuel J. Dunham

August 30, 2023

Paper

A Parametric Study of the SASI Comparing General Relativistic and Non-Relativistic Treatments*

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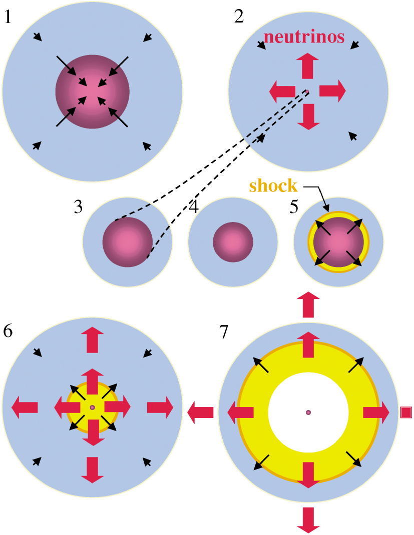
[5] *Department of Life and Physical Sciences, Fisk University, 1000 17th Ave N, Nasvhille, TN 37208*

Overview



Mezzacappa A. 2005.
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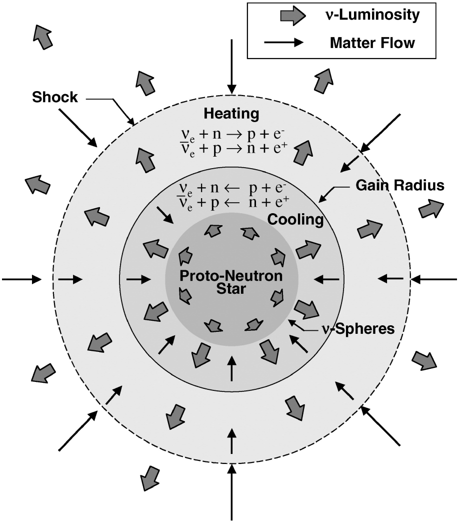

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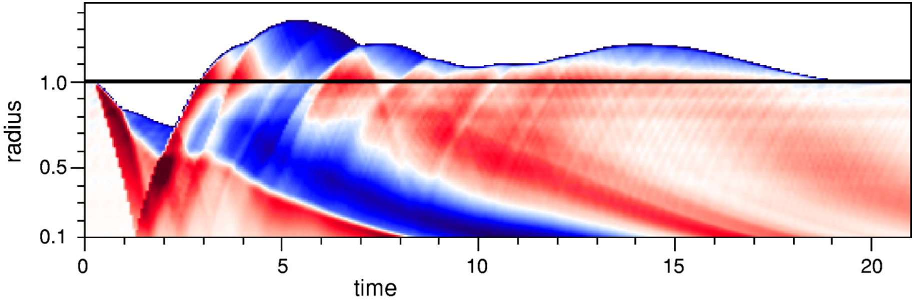
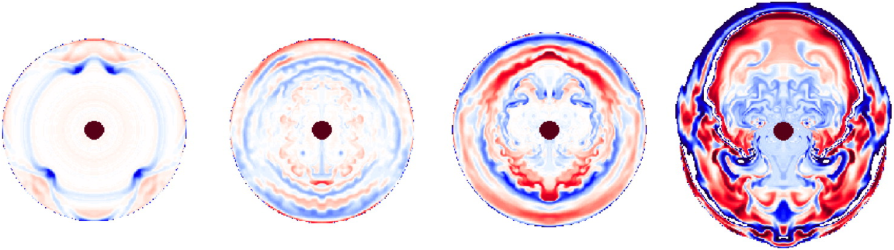


Figure: Equilibrium (white), under- (blue), and over- (red) pressure (Blondin et al., 2003).



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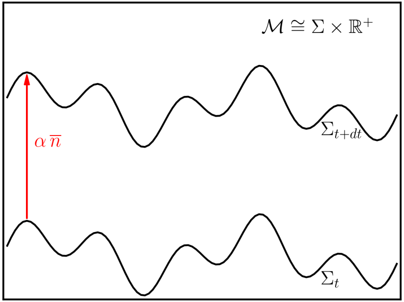
$d{+}1$ Decomposition

$$ds^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu$$



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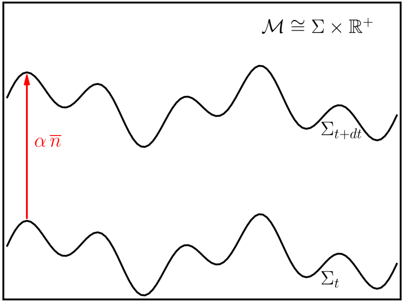

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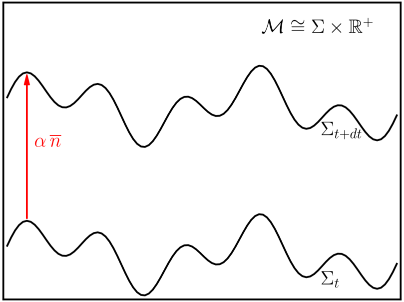

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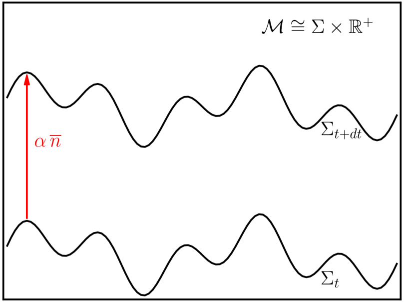

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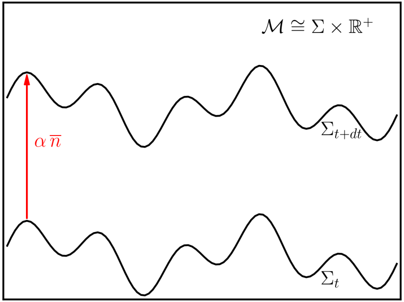
$0 < \alpha \leq 1$: Lapse Function



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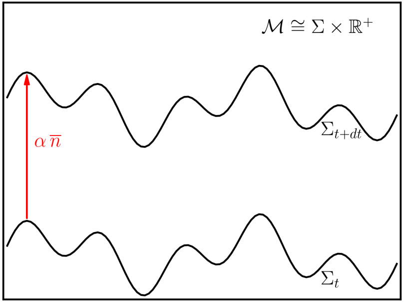
$\overline{n}$: Eulerian four-velocity

$$\left(\underline{\underline{g}}\,(\overline{n}, \overline{n}) = \overline{n} \cdot \overline{n} = -1\right)$$



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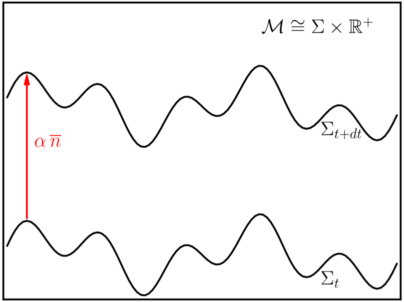
(Also, $\underline{\underline{K}}$: Extrinsic curvature)



Conformally-Flat Condition

Wilson et al. (1996); Cordero-Carrión et al. (2009)

$$\gamma_{ij} = \psi^4 \, \bar{\gamma}_{ij}$$

$1 \leq \psi < 2$: Conformal factor

$$\bar{\gamma}_{ij} = \mathrm{diag}\left(1, r^2, r^2 \sin^2 \theta\right)$$

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(Also, $K := \mathrm{Tr}_{\gamma_{ij}} \left(\underline{\underline{K}}\right) = \partial_t \, K = 0$)

Isotropic Coordinates ($c = G = 1$)

Baumgarte and Shapiro (2010)

$$\alpha\left(r\right) = \frac{1 - r_{\mathrm{Sc}}/r}{1 + r_{\mathrm{Sc}}/r}$$

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$$\beta^i = 0$$

$$K_{ij} = 0$$

Fluid Equations

Units defined such that $c = G = 1$

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$\overline{\overline{T}} := \rho \, h \, \overline{u} \otimes \overline{u} + p \, \overline{\overline{g}}$ (p: comoving thermal pressure,
$h := 1 + (e + p) / \rho$: specific enthalpy, e: comoving internal energy density)

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Five equations with six unknowns ☹

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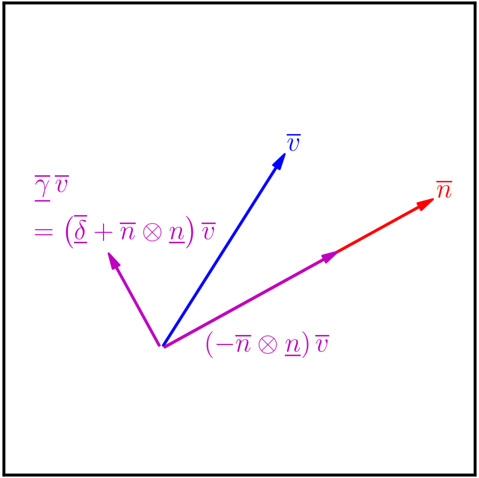
Five equations with six unknowns ☹

Close with an equation of state: $p = p\,(e) := (\Gamma - 1)\,e$, $\Gamma = 4/3$

Valencia Decomposition



Extensible to higher-rank tensors!

Valencia Decomposition

$$E := n_{\mu'} \, n_{\nu'} \, T^{\mu'\nu'}$$

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Math...

Valencia Decomposition

$$\partial_t \boldsymbol{U} + \frac{1}{\sqrt{\gamma}}\,\partial_i\left[\alpha\,\sqrt{\gamma}\,\boldsymbol{F}^i\left(\boldsymbol{U}\right)\right] = \boldsymbol{S}\left(\boldsymbol{U}\right)$$

GR

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$$\partial_t \boldsymbol{U} + \frac{1}{\sqrt{\gamma}} \partial_i \left[\alpha \sqrt{\gamma} \boldsymbol{F}^i (\boldsymbol{U}) \right] = \boldsymbol{S} (\boldsymbol{U})$$

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$\boldsymbol{U} := (D, S_j, \tau)^\top$

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NR

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$$\boldsymbol{F}^i (\boldsymbol{U}) = \begin{pmatrix} \rho W v^i \\ \rho h W^2 v^i v_j + p \delta^i{}_j \\ \left(\rho h W^2 - \rho W \right) v^i \end{pmatrix}$$

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$$\Phi (r) := -M/r$$

Steady-State Solutions (pre-shock)

$$\cancelto{0}{\partial_t \boldsymbol{U}} + \frac{1}{\sqrt{\gamma}}\,\partial_r\left[\alpha\,\sqrt{\gamma}\,\boldsymbol{F}^r\left(\boldsymbol{U}\right)\right] = \boldsymbol{S}\left(\boldsymbol{U}\right)$$

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$\alpha\,\psi^6\,W \times 4\pi\,r^2\,\rho\,v = -\dot{M}$

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Steady-State Solutions (pre-shock)

$$\cancelto{0}{\partial_t \boldsymbol{U}} + \frac{1}{\sqrt{\gamma}}\,\partial_r\left[\alpha\,\sqrt{\gamma}\,\boldsymbol{F}^r\left(\boldsymbol{U}\right)\right] = \boldsymbol{S}\left(\boldsymbol{U}\right)$$

GR

$$\alpha\,\psi^6\,W \times 4\pi\,r^2\,\rho\,v = -\dot{M}$$

$$\alpha\,h\,W = \mathcal{B}$$

$$p = K_1\,\rho^\Gamma$$

NR

$$4\pi\,r^2\,\rho\,v = -\dot{M}$$

Steady-State Solutions (pre-shock)

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NR

$$4\pi \, r^2 \, \rho \, v = -\dot{M}$$

$$\frac{1}{2} \, v^2 + h_{\mathrm{NR}} + \Phi = \mathcal{B}_{\mathrm{NR}}$$

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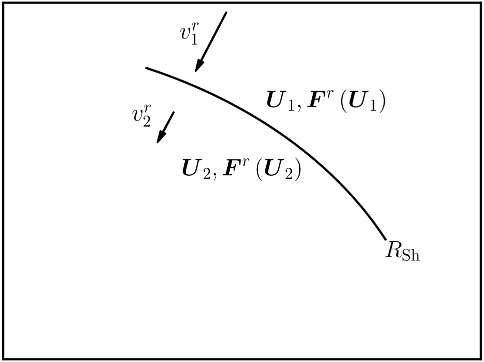
$p = K_1 \, \rho^\Gamma$

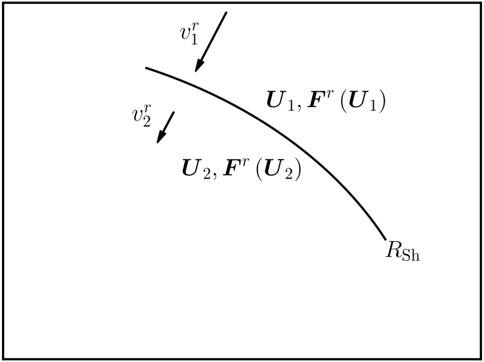
Jump Conditions

$\boldsymbol{U}_1 \neq \boldsymbol{U}_2$



Jump Conditions

$$\boldsymbol{U}_1 \neq \boldsymbol{U}_2$$
$$\boldsymbol{F}^r\left(\boldsymbol{U}_1\right) = \boldsymbol{F}^r\left(\boldsymbol{U}_2\right)$$



Jump Conditions

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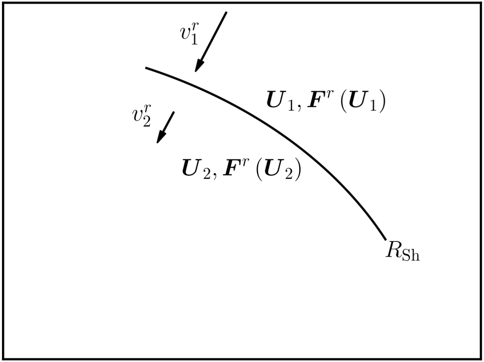
$\boldsymbol{F}^r(\boldsymbol{U}_1) = \boldsymbol{F}^r(\boldsymbol{U}_2)$

Yields:



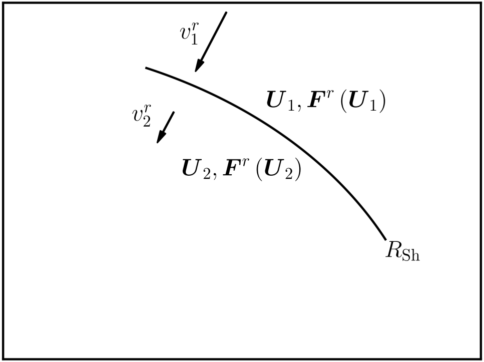
Jump Conditions

$\boldsymbol{U}_1 \neq \boldsymbol{U}_2$

$\boldsymbol{F}^r(\boldsymbol{U}_1) = \boldsymbol{F}^r(\boldsymbol{U}_2)$

Yields:

$\rho_2 > \rho_1$



Jump Conditions

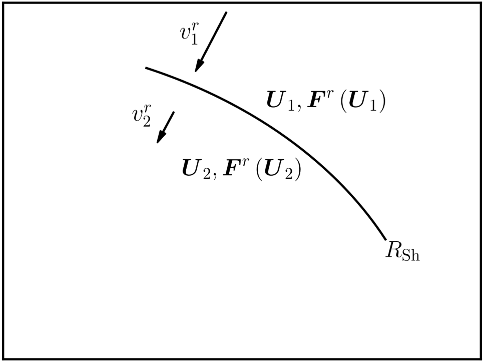
$\boldsymbol{U}_1 \neq \boldsymbol{U}_2$

$\boldsymbol{F}^r(\boldsymbol{U}_1) = \boldsymbol{F}^r(\boldsymbol{U}_2)$

Yields:

$\rho_2 > \rho_1$

$e_2(p_2) > e_1(p_1)$



Jump Conditions

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Yields:

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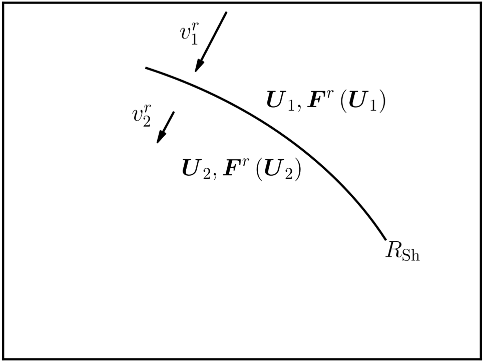
$e_2\left(p_2\right) > e_1\left(p_1\right)$

$K_2 > K_1$



Jump Conditions

$\boldsymbol{U}_1 \neq \boldsymbol{U}_2$

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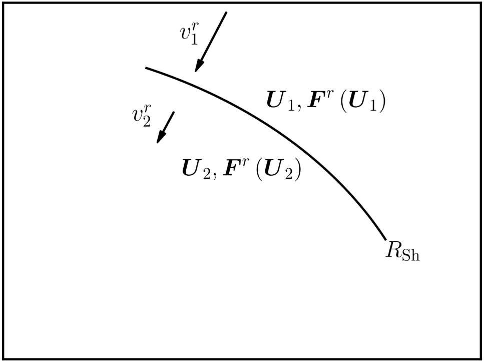
Yields:

$\rho_2 > \rho_1$

$e_2(p_2) > e_1(p_1)$

$K_2 > K_1$

$|v_2^r| < |v_1^r|$



Steady-State Solutions (post-shock)

$$\cancel{\partial_t \boldsymbol{U}}^{0} + \frac{1}{\sqrt{\gamma}} \partial_r \left[\alpha \sqrt{\gamma} \, \boldsymbol{F}^r \left(\boldsymbol{U} \right) \right] = \boldsymbol{S} \left(\boldsymbol{U} \right)$$

GR

$$\alpha \, \psi^6 \, W \times 4\pi \, r^2 \, \rho \, v = -\dot{M}$$

$$\alpha \, h \, W = \mathcal{B}$$

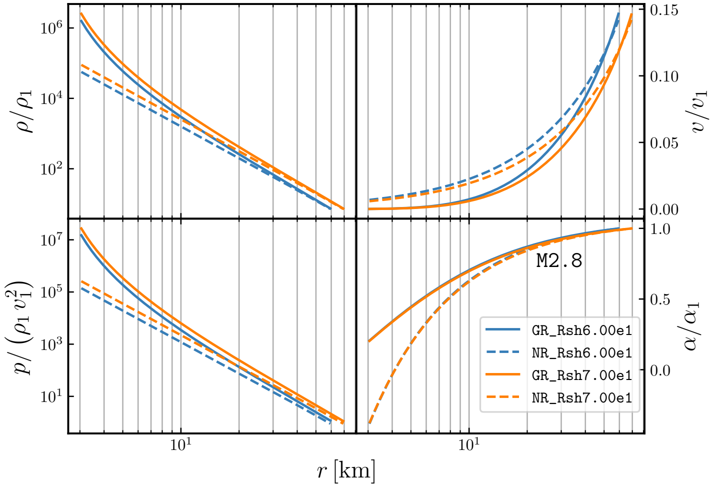
$$p = K_2 \, \rho^\Gamma$$

NR

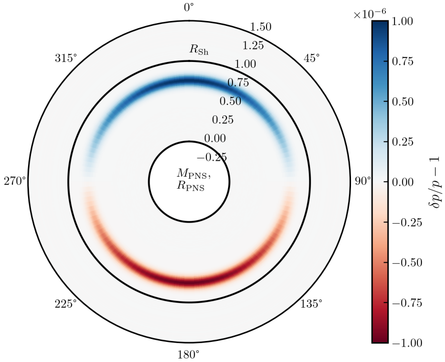
$$4\pi \, r^2 \, \rho \, v = -\dot{M}$$

$$\frac{1}{2} \, v^2 + h_{\mathrm{NR}} + \Phi = \mathcal{B}_{\mathrm{NR}}$$

$$p = K_2 \, \rho^\Gamma$$


10^6
10^4
10^2
ρ/ρ_1
0.15
0.10
0.05
0.00
v/v_1
10^7
10^5
10^3
10^1
$p/\left(\rho_1\, v_1^2\right)$
1.0
0.5
0.0
α/α_1
M2.8
GR_Rsh6.00e1
NR_Rsh6.00e1
GR_Rsh7.00e1
NR_Rsh7.00e1
10^1
10^1
r [km]

$$\eta(r) := \frac{r - R_{\mathrm{PNS}}}{R_{\mathrm{Sh}} - R_{\mathrm{PNS}}}$$



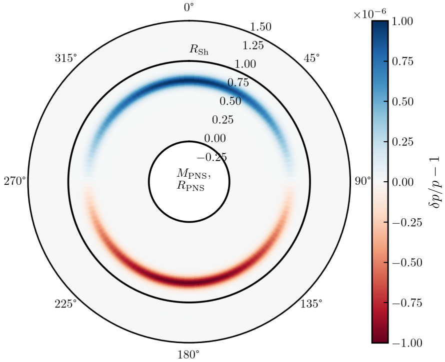
$$\eta\left(r\right) := \frac{r - R_{\mathrm{PNS}}}{R_{\mathrm{Sh}} - R_{\mathrm{PNS}}}$$

$$\frac{\delta p(\eta,\theta)}{p(\eta_{\mathrm{c}})} =$$

$$10^{-6} \times \exp\left[\frac{-(\eta - \eta_{\mathrm{c}})^2}{2\,\sigma^2}\right] \cos\theta$$

$$\eta_{\mathrm{c}} = 0.75$$

$$\sigma = 0.05$$



Parameter Space

Model parameters:
M_{PNS}, R_{PNS},
$R_{\mathrm{Sh}}\,(t=0)$, $\dot{M}$, K_1,
$\mathcal{B}_{(\mathrm{NR})}$, Γ

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Parameters we varied:
$\xi = (M_{\mathrm{PNS}}/M_{\odot})\,/\,(R_{\mathrm{PNS}}/20\,\mathrm{km})$ (O'Connor and Ott, 2011), $R_{\mathrm{Sh}}\,(t=0)$

Parameter Space

Model parameters:
M_{PNS}, R_{PNS}, $R_{\text{Sh}}\,(t=0)$, $\dot{M}$, K_1, $\mathcal{B}_{(\text{NR})}$, Γ

Parameters we varied:
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Table 1. Model Parameters

Model	M_{PNS} [$M_{\odot}$]	R_{PNS} [km]	R_{sh} [km]	ξ
`M1.4_Rpns040_Rsh1.20e2`	1.4	40	120	0.7
`M1.4_Rpns040_Rsh1.50e2`	1.4	40	150	0.7
`M1.4_Rpns040_Rsh1.75e2`	1.4	40	175	0.7
`M2.8_Rpns020_Rsh6.00e1`	2.8	20	60	2.8
`M2.8_Rpns020_Rsh7.00e1`	2.8	20	70	2.8

Note—Model parameters chosen for the 5 models. All models were run with both GR and NR. The first three rows correspond to the low-compactness models and the last two rows correspond to the high-compactness models.

$A := \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(v^{\theta}\ \sin\theta\right)$ (Scheck et al., 2008) `<movie>`

$A := \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(v^{\theta}\,\sin\theta\right)$ (Scheck et al., 2008) <movie>

$$A\left(r,\theta,t\right) = \sum_{\ell'=0}^{\infty} G_{\ell'}\left(r,t\right) P_{\ell'}\left(\cos\theta\right)$$

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$$A\left(r,\theta,t\right) = \sum_{\ell'=0}^{\infty} G_{\ell'}\left(r,t\right) P_{\ell'}\left(\cos\theta\right)$$

$$\implies G_{\ell}\left(r,t\right) := \frac{1}{N_{\ell}}\int_{0}^{\pi} A\left(r,\theta,t\right)\,P_{\ell}\left(\cos\theta\right)\,\sin\theta\,d\theta$$

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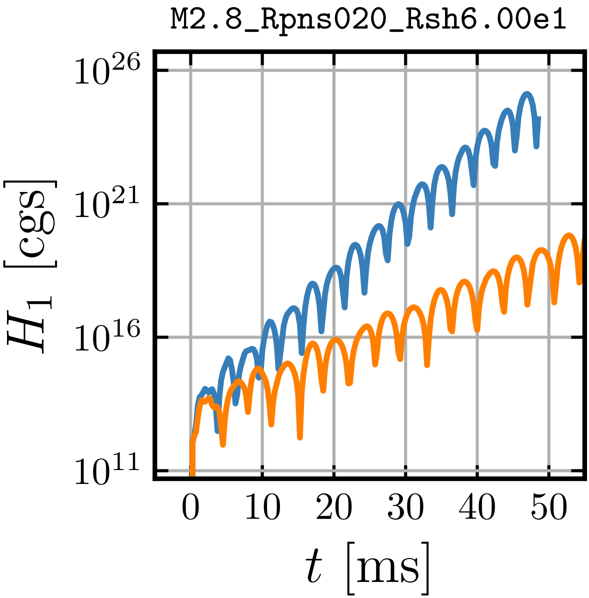
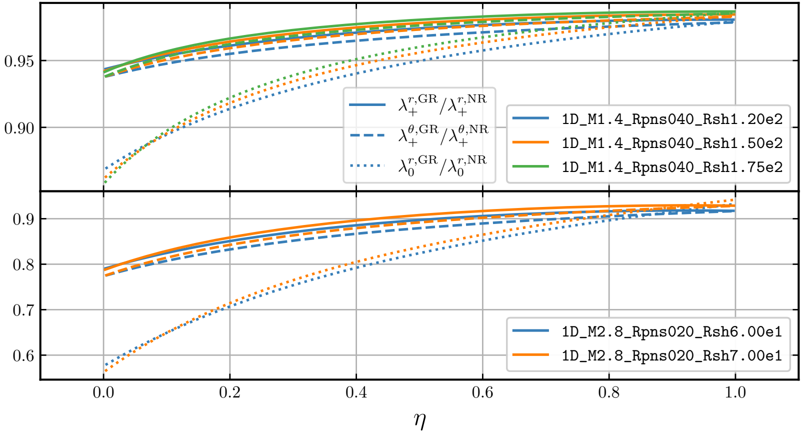
$H_{\ell}\left(t\right) := 4\pi \int_{r_a}^{r_b} \left[G_{\ell}\left(r,t\right)\right]^{2} \left[\psi\left(r\right)\right]^{6} r^{2}\, dr$ <movie>

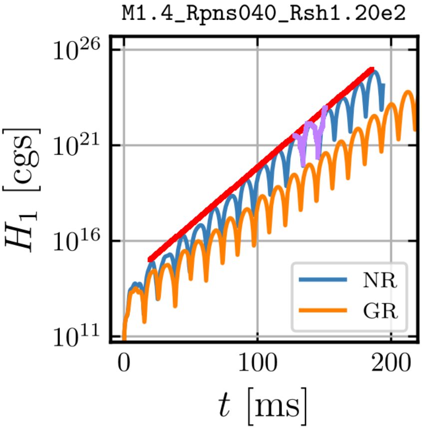


M2.8_Rpns020_Rsh6.00e1
10^{26}
10^{21}
10^{16}
10^{11}
H_1 [cgs]
0
10
20
30
40
50
t [ms]


0.95
0.90
0.9
0.8
0.7
0.6
0.0
0.2
0.4
0.6
0.8
1.0
η
$\lambda_+^{r,\mathrm{GR}}/\lambda_+^{r,\mathrm{NR}}$
$\lambda_+^{\theta,\mathrm{GR}}/\lambda_+^{\theta,\mathrm{NR}}$
$\lambda_0^{r,\mathrm{GR}}/\lambda_0^{r,\mathrm{NR}}$
1D_M1.4_Rpns040_Rsh1.20e2
1D_M1.4_Rpns040_Rsh1.50e2
1D_M1.4_Rpns040_Rsh1.75e2
1D_M2.8_Rpns020_Rsh6.00e1
1D_M2.8_Rpns020_Rsh7.00e1

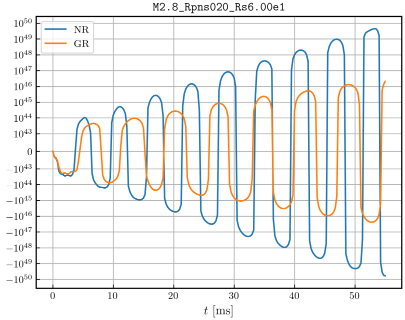
$$F(t) = F(0)\, e^{2\omega t} \sin^2\left(\frac{2\pi t}{T} + \delta\right)$$

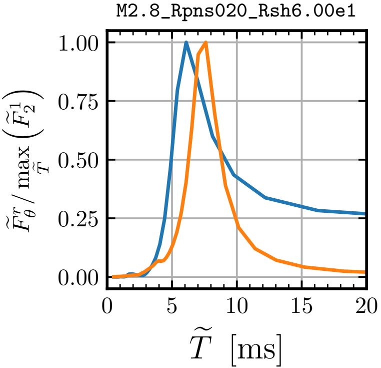
(Blondin and Mezzacappa, 2006)



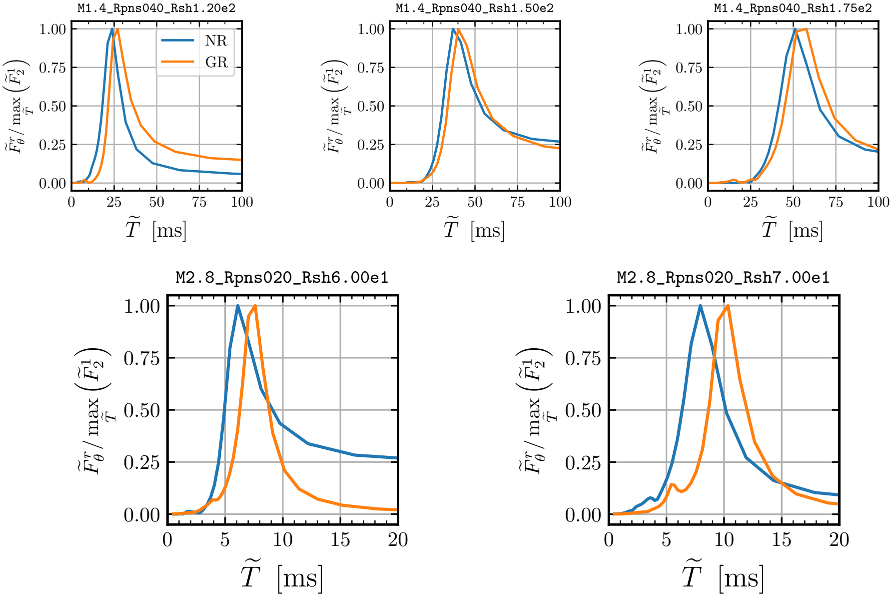
$$F^r_\theta := \alpha\, \psi^6\, h\, W^2 \times \sqrt{\bar{\gamma}}\, \rho\, v^r\, v_\theta$$

$$\widetilde{F}^r_\theta = \mathrm{FFT}\left\{F^r_\theta\right\}$$

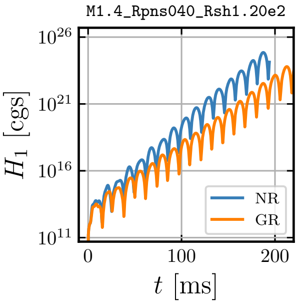


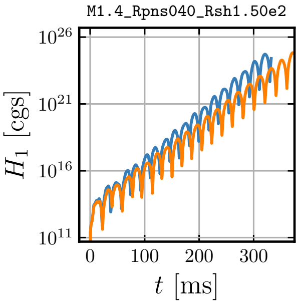


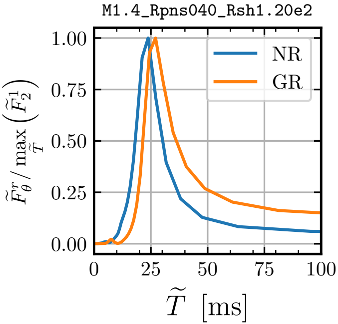
T defined as the unique $\widetilde{T}$ such that $\widetilde{F}^r_\theta\left(\widetilde{T}\right) = 1$

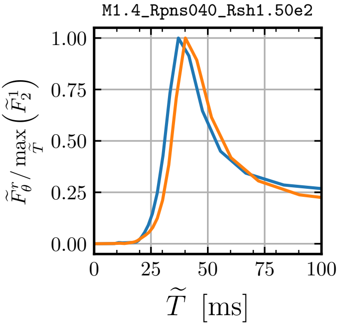


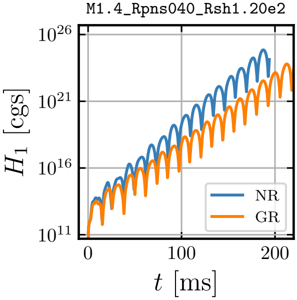
M1.4_Rpns040_Rsh1.20e2
M1.4_Rpns040_Rsh1.50e2
M1.4_Rpns040_Rsh1.75e2
M2.8_Rpns020_Rsh6.00e1
M2.8_Rpns020_Rsh7.00e1
NR
GR
$\widetilde{F}^r_\theta / \max_{\widetilde{T}} \left(\widetilde{F}^1_2 \right)$
$\widetilde{T}$ [ms]

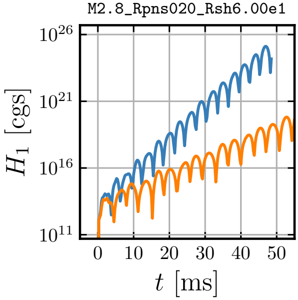

M1.4_Rpns040_Rsh1.20e2
10^{26}
10^{21}
10^{16}
10^{11}
H_1 [cgs]
0
100
200
t [ms]
NR
GR

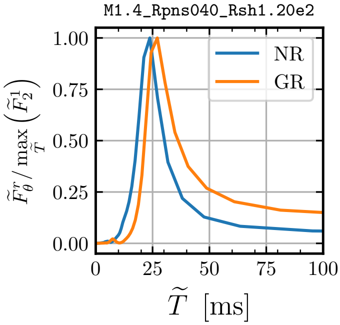

M1.4_Rpns040_Rsh1.50e2
10^{26}
10^{21}
10^{16}
10^{11}
H_1 [cgs]
0
100
200
300
t [ms]

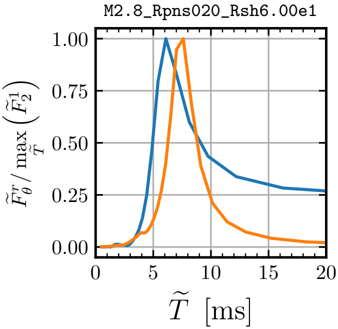

M1.4_Rpns040_Rsh1.20e2
1.00
0.75
0.50
0.25
0.00
$\widetilde{F}^r_\theta / \max_{\widetilde{T}} \left(\widetilde{F}^1_2\right)$
0
25
50
75
100
$\widetilde{T}$ [ms]
NR
GR

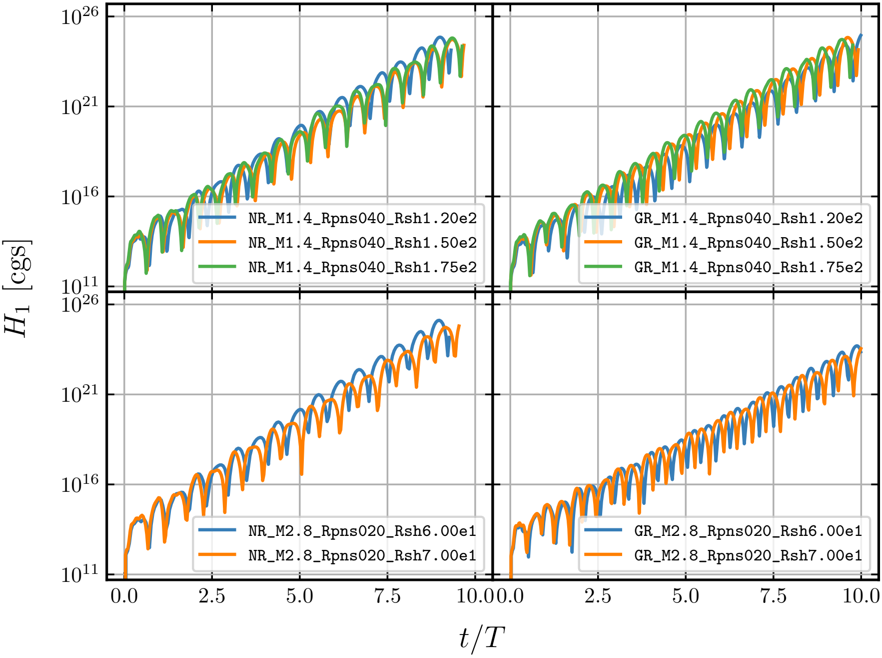

M1.4_Rpns040_Rsh1.50e2
1.00
0.75
0.50
0.25
0.00
$\widetilde{F}^r_\theta / \max_{\widetilde{T}} \left(\widetilde{F}^1_2\right)$
0
25
50
75
100
$\widetilde{T}$ [ms]


M1.4_Rpns040_Rsh1.20e2
10^{26}
10^{21}
10^{16}
10^{11}
H_1 [cgs]
0
100
200
t [ms]
NR
GR


M2.8_Rpns020_Rsh6.00e1
10^{26}
10^{21}
10^{16}
10^{11}
H_1 [cgs]
0
10
20
30
40
50
t [ms]

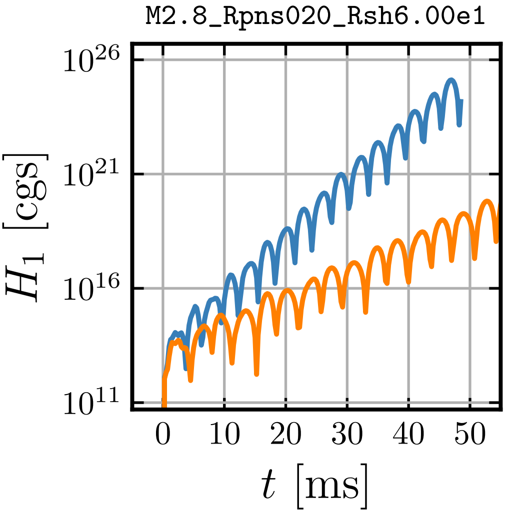

M1.4_Rpns040_Rsh1.20e2
1.00
0.75
0.50
0.25
0.00
$\widetilde{F}^r_\theta \,/\, \max_{\widetilde{T}} \left(\widetilde{F}^1_2\right)$
0
25
50
75
100
$\widetilde{T}$ [ms]
NR
GR


M2.8_Rpns020_Rsh6.00e1
1.00
0.75
0.50
0.25
0.00
$\widetilde{F}^r_\theta \,/\, \max_{\widetilde{T}} \left(\widetilde{F}^1_2\right)$
0
5
10
15
20
$\widetilde{T}$ [ms]

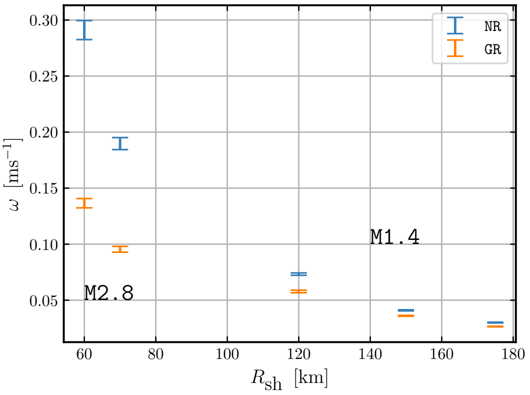


10^{26}
10^{21}
10^{16}
10^{11}
H_1 [cgs]
NR_M1.4_Rpns040_Rsh1.20e2
NR_M1.4_Rpns040_Rsh1.50e2
NR_M1.4_Rpns040_Rsh1.75e2
GR_M1.4_Rpns040_Rsh1.20e2
GR_M1.4_Rpns040_Rsh1.50e2
GR_M1.4_Rpns040_Rsh1.75e2
NR_M2.8_Rpns020_Rsh6.00e1
NR_M2.8_Rpns020_Rsh7.00e1
GR_M2.8_Rpns020_Rsh6.00e1
GR_M2.8_Rpns020_Rsh7.00e1
0.0
2.5
5.0
7.5
10.0
t/T

Conclusions



Conclusions



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Summary

- Extended study of Blondin and Mezzacappa (2006) to include GR
- Showed that GR leads to longer SASI oscillation period than NR
- Showed that GR leads to smaller SASI growth rate than NR
- Found that growth rate is such that ωT is roughly constant for some parameter sets: implications for growth rate mechanism
- Future Work
    - Extend study to 3D
    - Include GR monopole (Marek et al., 2006)